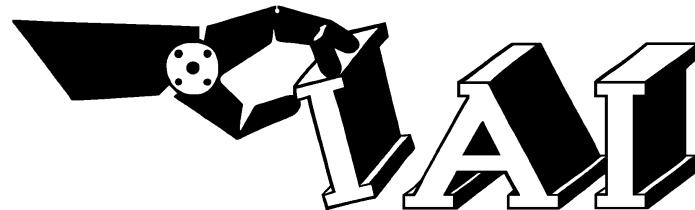


Intro to Avalanche Theory

Dr. Philip Goetz

Intelligent Automation, Inc.

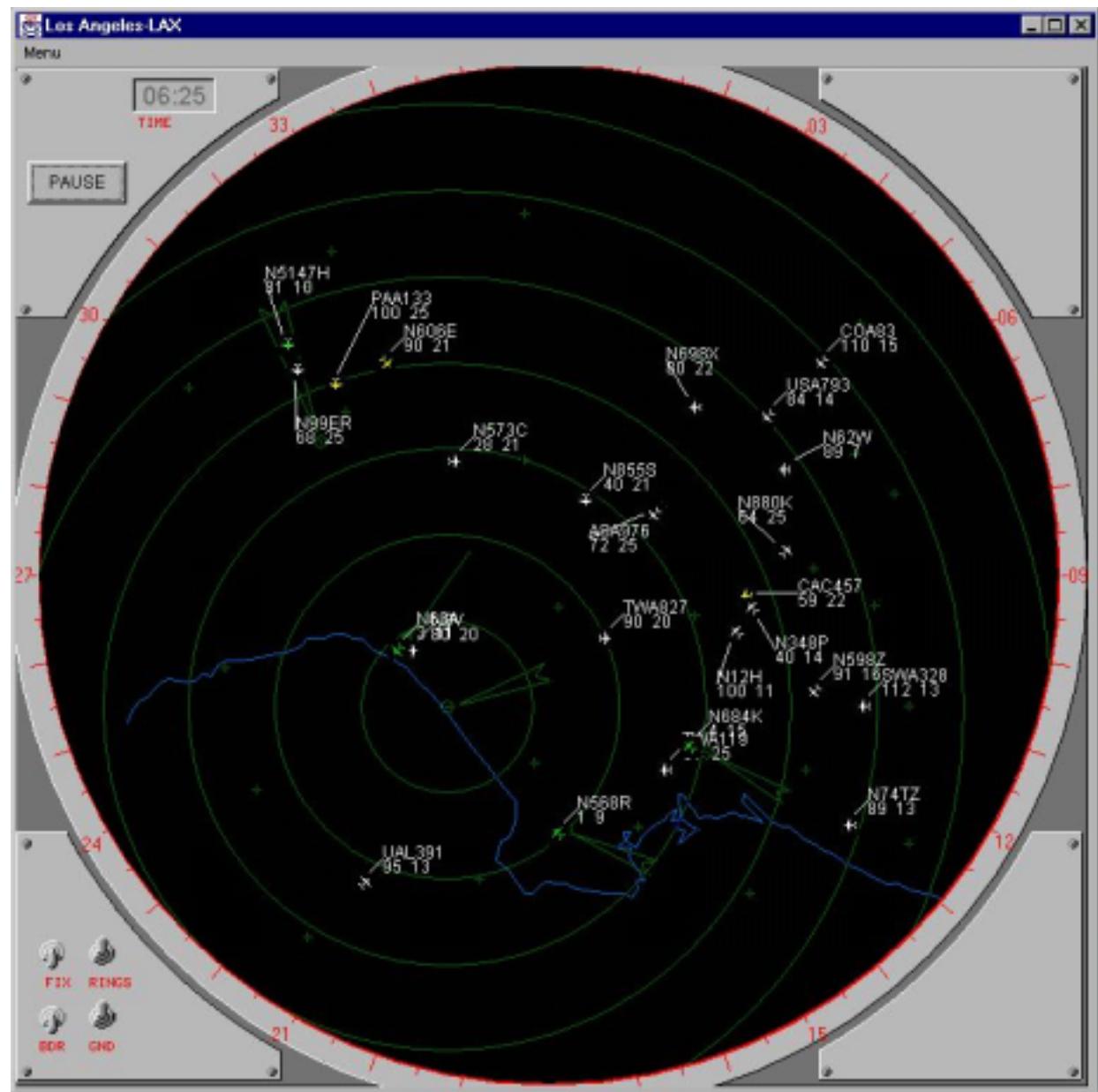


NASA Distributed Air Ground Workshop, May 24 2000

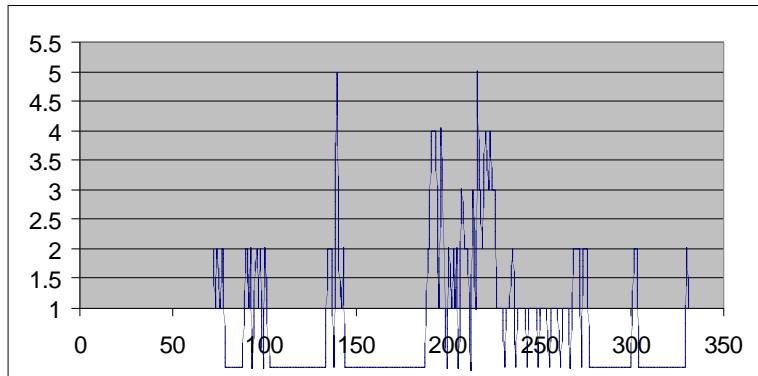
Phases of Systems

“System”: A state, and rules that say how the current state determines the next state.

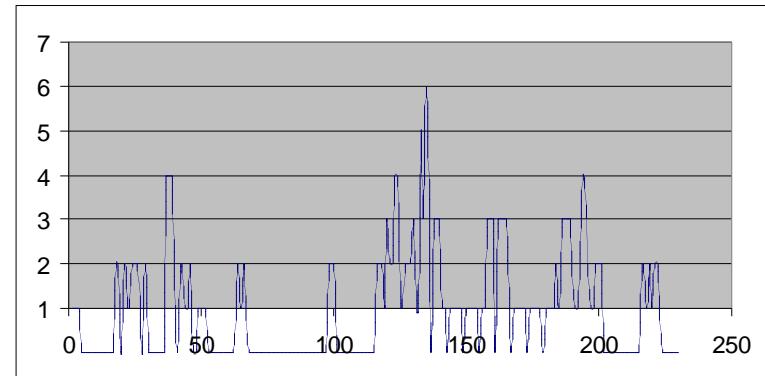
- **Stable**: Solid, frozen. Independent events with Poisson distribution.
- **Unstable**: Liquid, gaseous. Burst sizes may have uniform (white-noise) distribution.
- **Semi-stable, critically stable**:
Spin glasses, crystals.
Burst size has power-law or 1/f distribution.



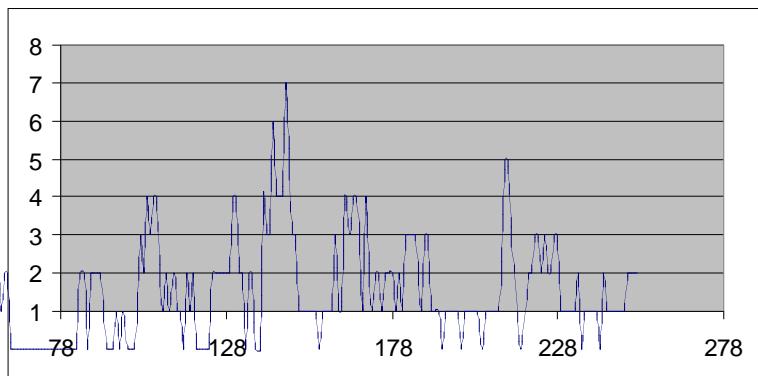
Evasive actions vs. time



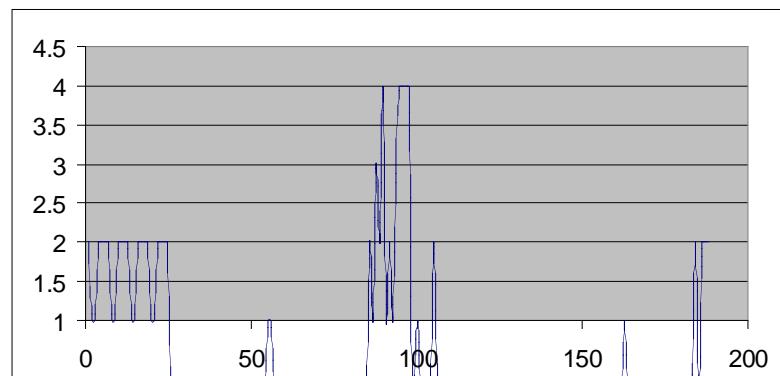
15 aircraft



20 aircraft



25 aircraft



30 aircraft

NOT a Poisson distribution.

So What?

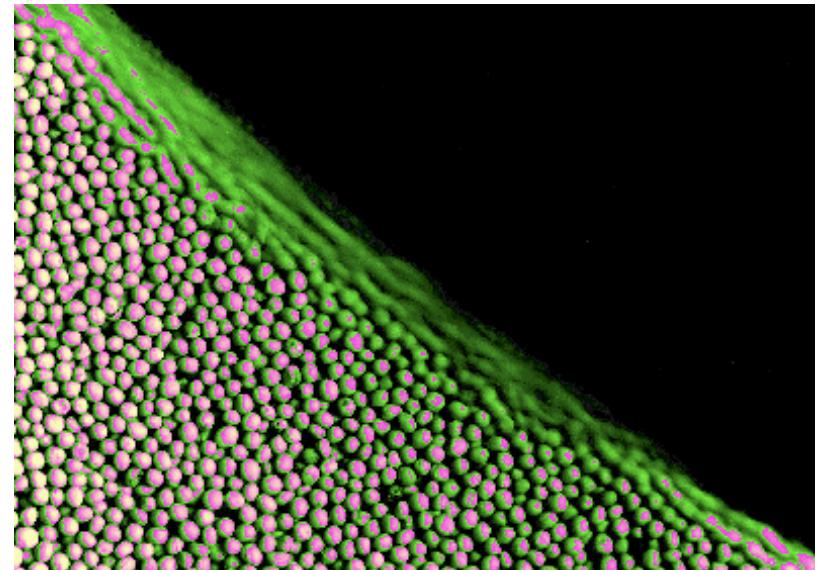


Examples of Semi-stable Systems

- Discrete event networks
 - Sandpiles (Bak)
 - Cellular automata (Wolfram, Langton)
 - Percolation lattices (Stauffer)
 - Boolean networks (Kauffman)
 - Recurrent neural networks (Hopfield)
- Continuous systems with chaotic trajectories

Sandpiles

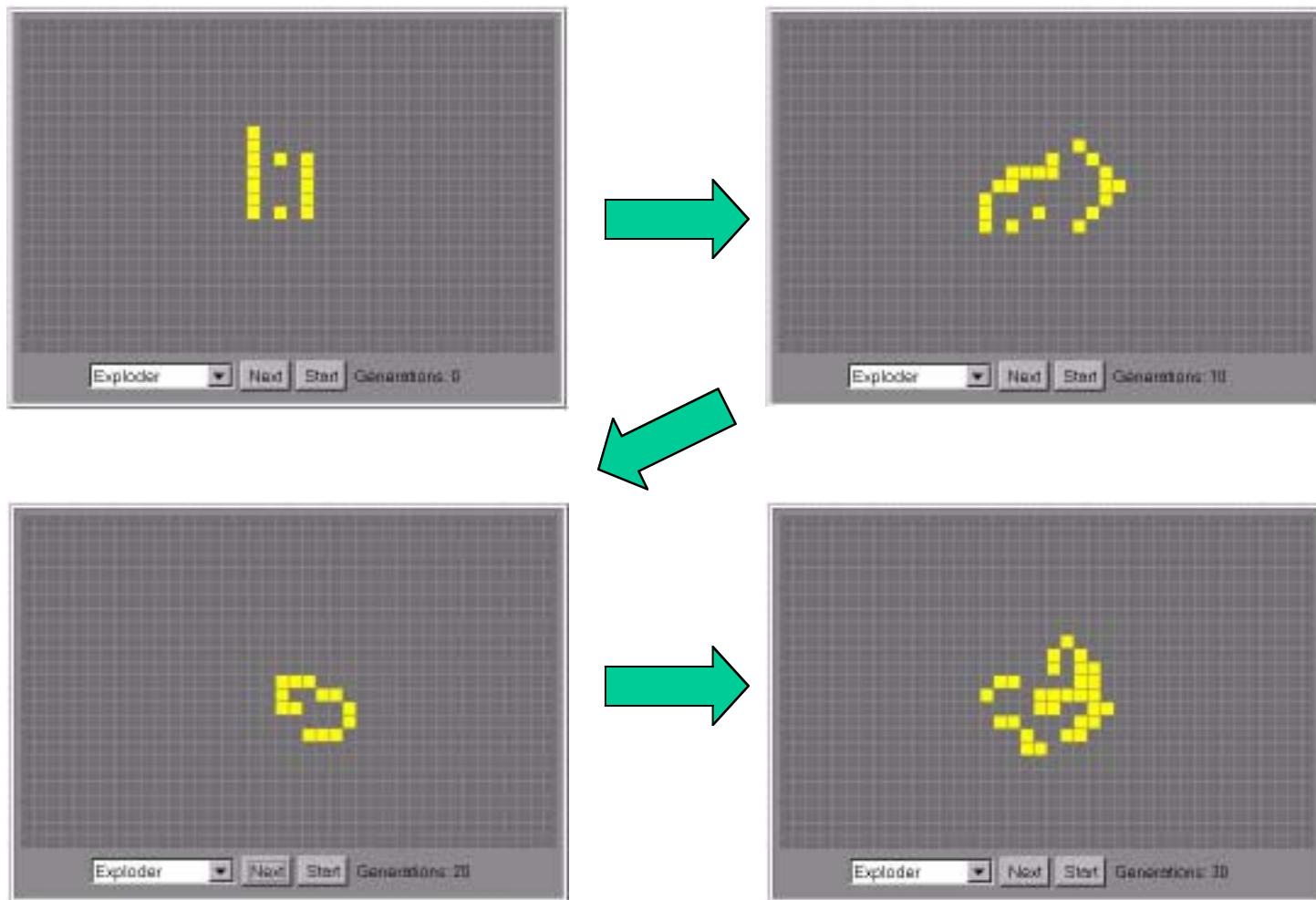
- Power-law distribution of avalanche sizes
- Order parameter: Slope
- Similar results for earthquake sizes, extinction events



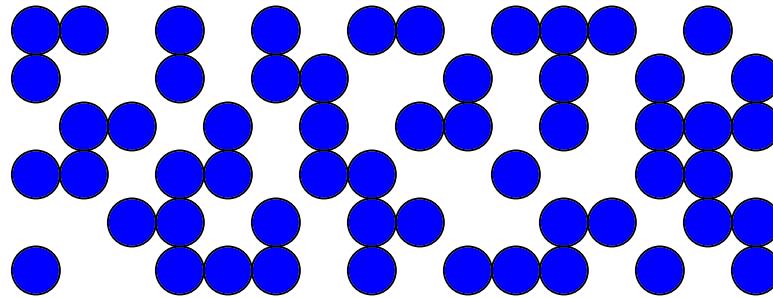
Cellular automata

- Wolfram's classification of CA rule sets
 - Type I: Static (fixed point)
 - Type II: Periodic (limit cycle)
 - Type III: Random, divergent
 - Type IV: Complex (Conway's game of Life)
- Langton's order parameter: $\lambda = (K^N - n_q) / K^N$
 - $K \geq 4, N \geq 5, \lambda_c = .5$ for certain CAs
 - Power-law distribution of avalanche sizes at λ_c

John Conway's Game of Life



Percolation lattices



- Order parameter: Lattice density p
 - $p_c = .59275$ for square lattice
 - $p_c = .5$ for bond lattice (20 years to prove)
 - Infinite grid is connected iff $p > p_c$
- Power-law cluster size distribution at p_c
 - $n(s) \sim s^{-187/91}$ for square lattice

Boolean networks

- Order parameter: $K = \text{ave. number of inputs}$
- Critical value: $K = 2$
 - $K < 2$: Short transients
 - $K = 2$: Transients of all sizes
 - $K > 2$: Long transients

Generalizations

- Semi-stability depends on nonlinearity
- Cannot predict outcome of a given event
- Distribution of “event burst” sizes:
 - Stable system: Poisson distribution: $n(s) \sim (e^{-\mu} \mu^s)/s!$
 - Critical system: Power-law distribution: $n(s) \sim s^{-c}$
- Order parameter can tell which regime (stable, critical, unstable) a system is in

What if airspace is semi-stable?

- Bad news:
 - Stable domain can be close to unstable domain
 - No way of predicting a burst's size in semi-stable phase
 - Can't distinguish semi-stable phase from other phases except by long-term patterns
- Good news:
 - Order parameter = stability metric
 - Power-law distributions \Rightarrow critical stability

Applications

- Stability could be used to test
 - URET: diversions around hot sectors
 - FSM: suggest need for ground delay
 - Miles in trail propagation
 - rerouting playbook
 - Dynamic sectorization: sector shapes

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